Quantum Simulation: from Feynman's Vision to Today's Reality and into the Future

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Richard Feynman's
Centennial Celebration
Université de Geneve
November 30, 2018

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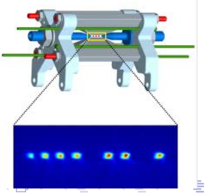


Outline

A Brief History of Computing

Classical Simulation of Quantum Systems?

From Feynman's Vision to Today's Reality

From Wilson's Lattice QCD to Quantum Link Models

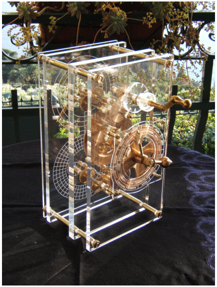
Quantum Simulators for Gauge Theories

With Feynman's Inspiration into the Future

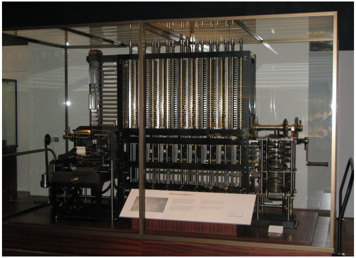
Outline

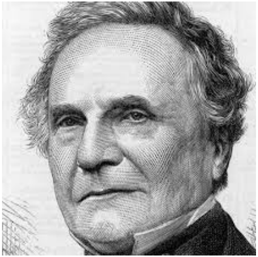
The first “digital computer” in Babylonia about 2400 b.c.

The first “analog computer”: Antikythera for determining the position of celestial bodies, Crete, about 100 b.c.



The first programmable computer: Charles Babbagge's (1791-1871) "difference engine" was realized by his son.





The first software developer: Ada Lovelace (1815-1852).

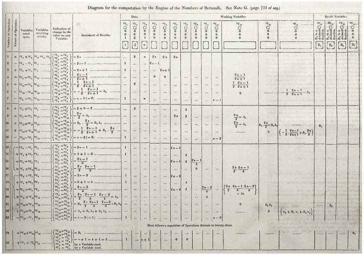
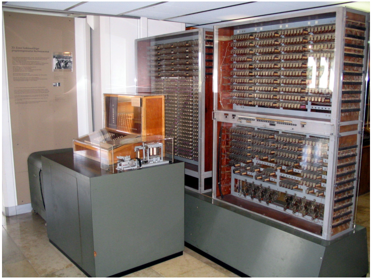
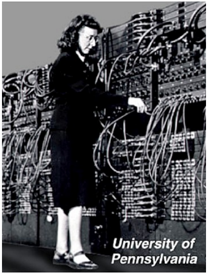


Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 et seq.)

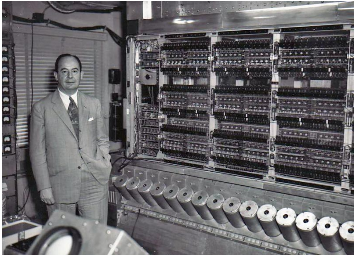
Konrad Zuse's (1910-1992) relay-driven computer Z3

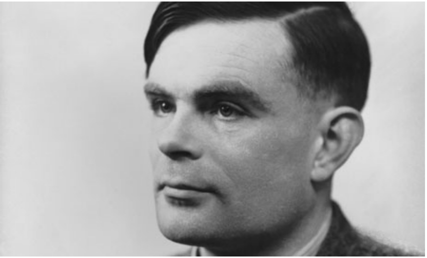


From the vacuum-tube ENIAC to the IBM Blue Gene

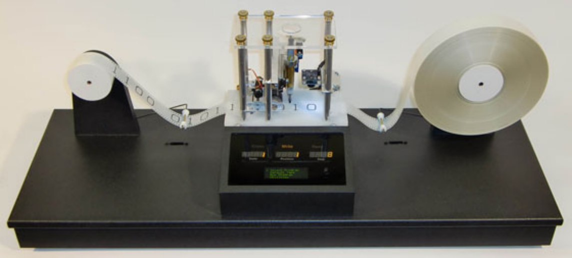


Pioneers of theoretical computer science: John von Neumann (1903-1957) and Alan Turing (1912-1954)





Model of a universal Turing machine



RSA encryption: multiplication is easy, factorization is hard.

RSA decryption challenge in 1991:
factorize the following 174-digit number with 576 bits

$$
\begin{aligned}
RSA576 \quad = \quad & 188198812920607963838697239461650439807163563 \\
& 379417382700763356422988859715234665485319 0 \\
& 60606504743045317388011303396716199692321205 \\
& 734031879550656996221305168759307650257059
\end{aligned}
$$

RSA encryption: multiplication is easy, factorization is hard.

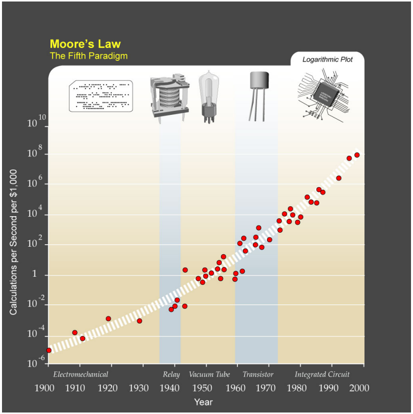
RSA decryption challenge in 1991:
factorize the following 174-digit number with 576 bits

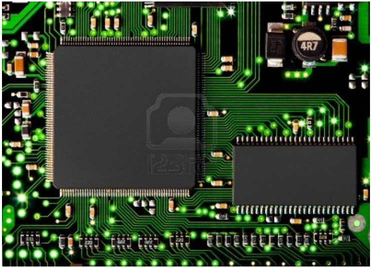
$$
\begin{aligned}
RSA576 \;=\;& 1881988129206079638386972394616504398071635 6\\
& 3379417382700763356422988859715234665485319 0\\
& 6060650474304531738801130339671619969232120 5\\
& 7340318795506569962213051687593076502570 59\\
=\;& {\color{red}3980750864240649373971255005503864911990643 6}\\
& {\color{red}2342526708406385189575946388957261768583317}\\
*\;& {\color{green}4727721461074353025362230719730482246329146 9}\\
& {\color{green}5302097116459852171130520711256363590397527}
\end{aligned}
$$

This problem was solved only in 2003 by two mathematicians in Bonn using very large computer resources.

Only in 2009, when the challenge was no longer active, the 232-digit number RSA768 with 768 bits has finally been factorized.

Moore's law: "Every two years the number of transistors per area increases by a factor of 2."





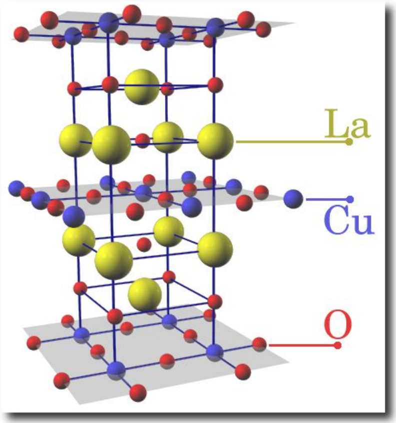
Modern micro chips consist of several billions of transistors, each about 10^{-8} m in size. This is already close to the quantum mechanical limit set by the size of individual atoms.

Outline

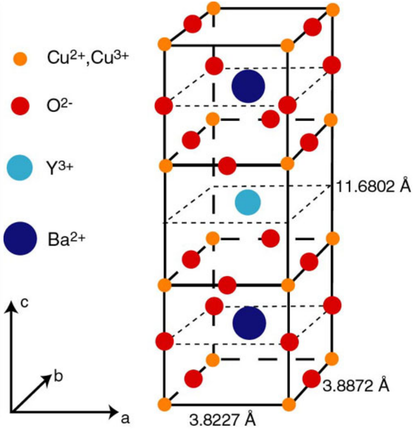
Richard Feynman's vision of 1982
(Int. J. Theor. Phys. 21 (1982) 467)

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

High-T_c superconducting materials

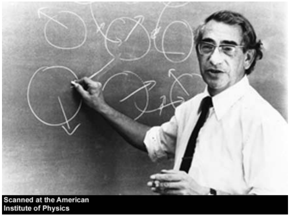


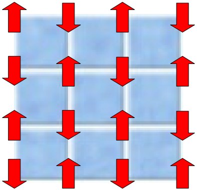
LaCuO



YBaCuO

The Hubbard Model for doped antiferromagnets





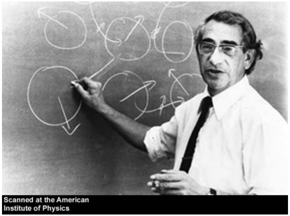
$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

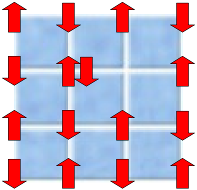
reduces to the Heisenberg model at half-filling for $U \gg t$

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
Does the Hubbard model explain high-T_c superconductivity?

The Hubbard Model for doped antiferromagnets





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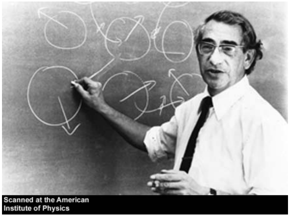

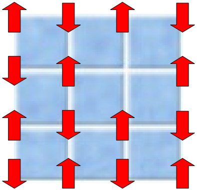

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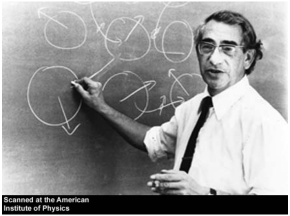

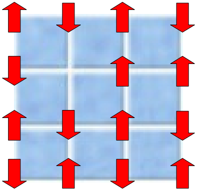

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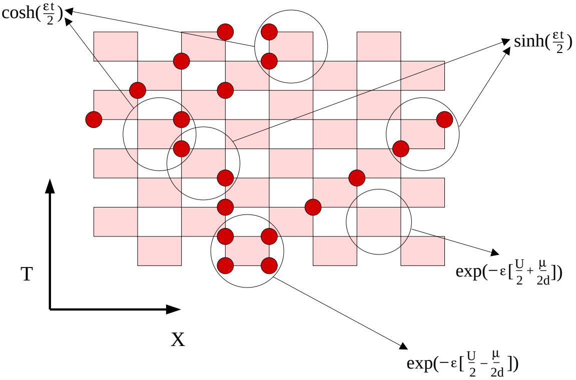
Important open question:
Does the Hubbard model explain high-T_c superconductivity?

Richard Feynman's vision of 1982

"Can quantum systems be probabilistically simulated by a classical computer? This is the hidden variable problem: it is impossible to represent the results of quantum mechanics with a classical universal device."

Path integral

$$
\begin{aligned}
Z_f &= \mathrm{Tr}[\exp(-\varepsilon H_1)\exp(-\varepsilon H_2)\ldots\exp(-\varepsilon H_M)]^N \\
&= \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])
\end{aligned}
$$



Sign problem of fermionic path integrals

$$Z_f = \mathrm{Tr}\exp(-\beta H) = \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n]) \ , \quad \mathrm{Sign}[n] = \pm 1$$

Average sign is exponentially small

$$\langle \mathrm{Sign} \rangle = \frac{\sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])}{\sum_{[n]} \exp(-S[n])} = \frac{Z_f}{Z_b} = \exp(-\beta V \Delta f)$$

The statistical error is exponentially large

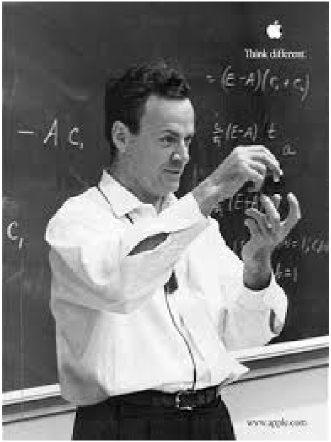
$$\frac{\sigma_{\mathrm{Sign}}}{\langle \mathrm{Sign} \rangle} = \frac{\sqrt{\langle \mathrm{Sign}^2 \rangle - \langle \mathrm{Sign} \rangle^2}}{\sqrt{N}\langle \mathrm{Sign} \rangle} = \frac{\exp(\beta V \Delta f)}{\sqrt{N}} \ .$$

Some very hard sign problems are NP complete

M. Troyer, UJW, Phys. Rev. Lett. 94 (2005) 170201.

Outline

Richard Feynman's vision of 1982



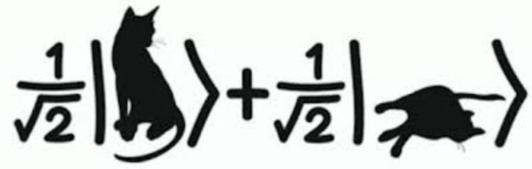
"Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws. Its not a Turing machine, but a machine of a different kind."

From bits to qubits

$$|\Psi\rangle = a|1\rangle + b|0\rangle, \quad |a|^2 + |b|^2 = 1$$

Entangled state of two qubits

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)$$

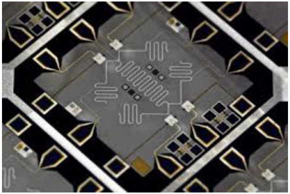


A universal quantum computer (David Deutsch's quantum analog of a classical Turing machine) could use Peter Shor's algorithm to solve the factorization problem.

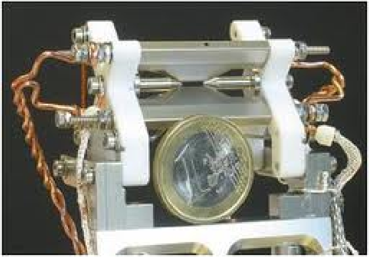
David Deutsch

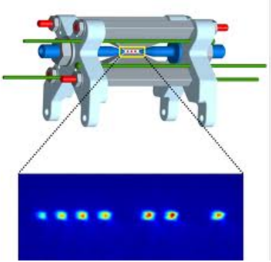
Peter Shor

Until today, only $15 = 3 \cdot 5$ has been correctly factorized by a quantum computer, at least in about 50 % of all trials.



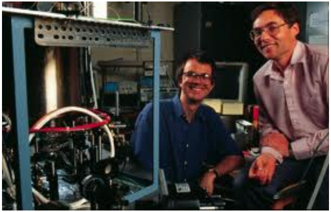
Ion traps as a digital quantum computer?



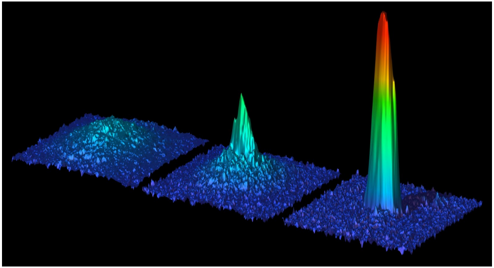


Franklin Medal 2010: I. Cirac, D. Wineland, P. Zoller

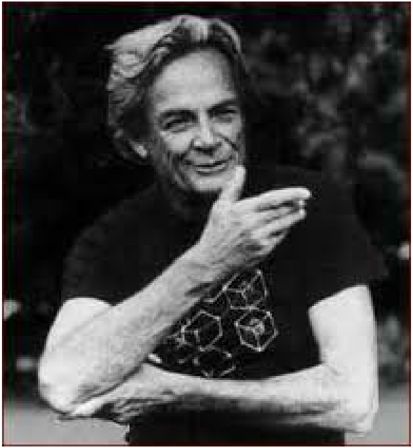
Bose-Einstein condensation in ultra-cold atomic gases

Eric Cornell, Carl Wieman, Wolfgang Ketterle, 1995

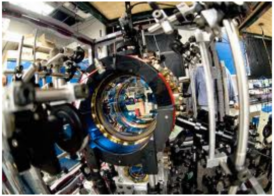


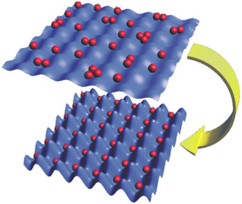
Richard Feynman's vision of 1982

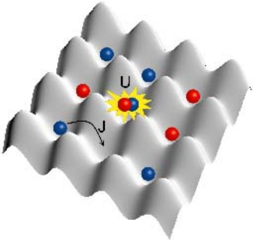


"I believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world."

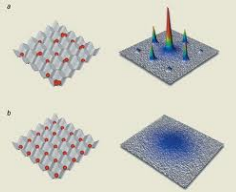
Ultra-cold atoms in optical lattices as analog quantum simulators



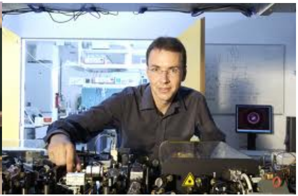




Transition from a superfluid to a Mott insulator



Theodor Hänsch

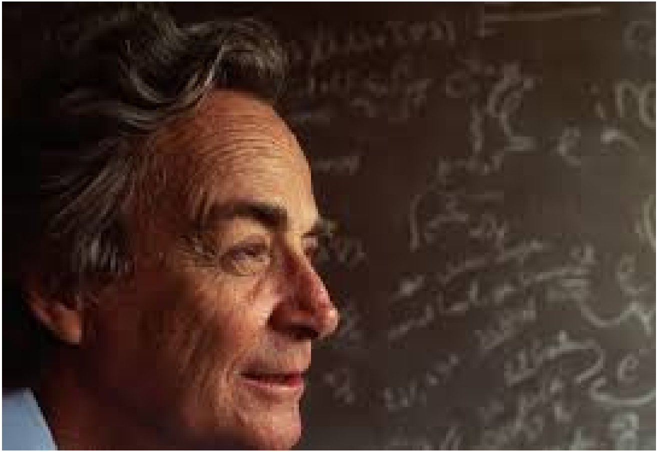


Immanuel Bloch

Can one understand high-T_c superconductivity in this way?

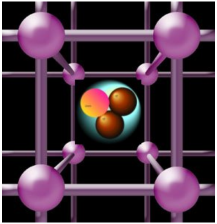
Outline

Richard Feynman's vision of 1982

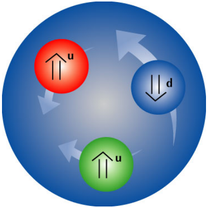


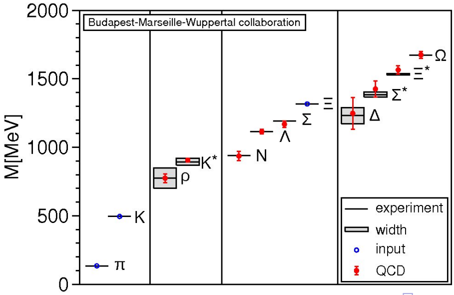
"It does seem to be true that all the various field theories have the same kind of behavior, and can be simulated in every way, apparently, with little latticeworks of spins and other things."

Kenneth Wilson's lattice QCD describes confinement of quarks and gluons inside protons and neutrons

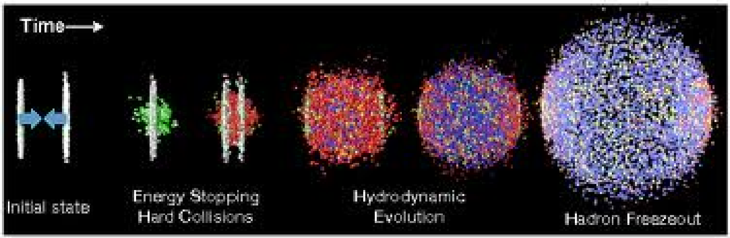


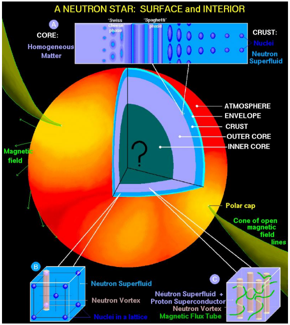
and confirms the experimentally measured mass spectrum

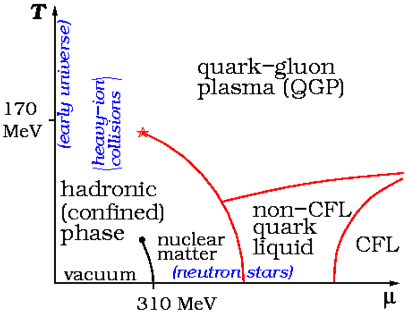




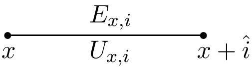
Can heavy-ion collision physics or nuclear astrophysics benefit from quantum simulations in the long run?





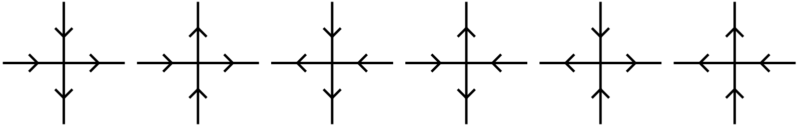


$U(1)$ gauge fields from spins $\frac{1}{2}$

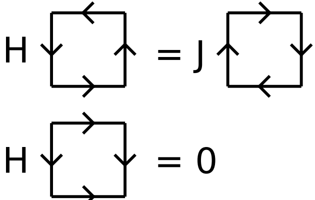


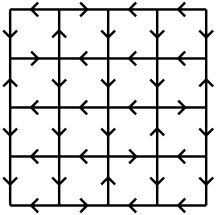
$$U = S^+, \; U^\dagger = S^-, \; E = S^3.$$

Gauss law



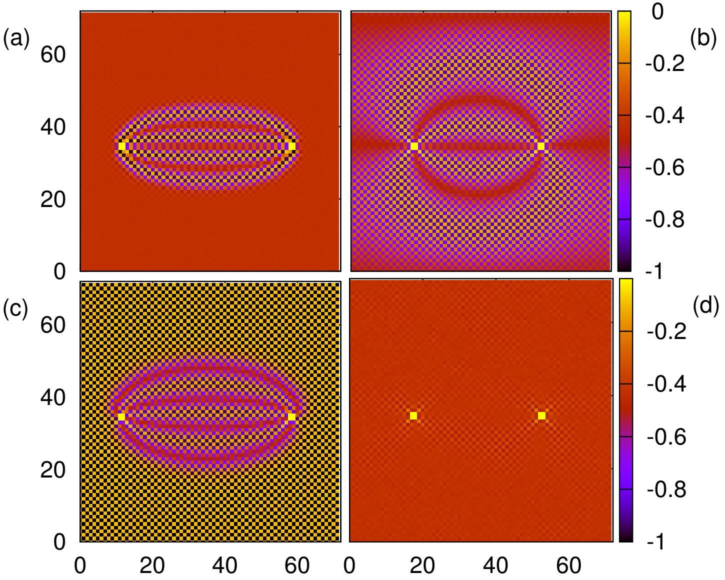
Ring-exchange plaquette Hamiltonian





D. Horn, Phys. Lett. B100 (1981) 149
P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

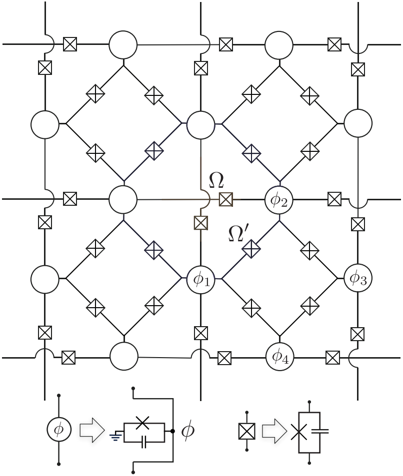
Energy density of charge-anti-charge pair $Q = \pm 2$



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

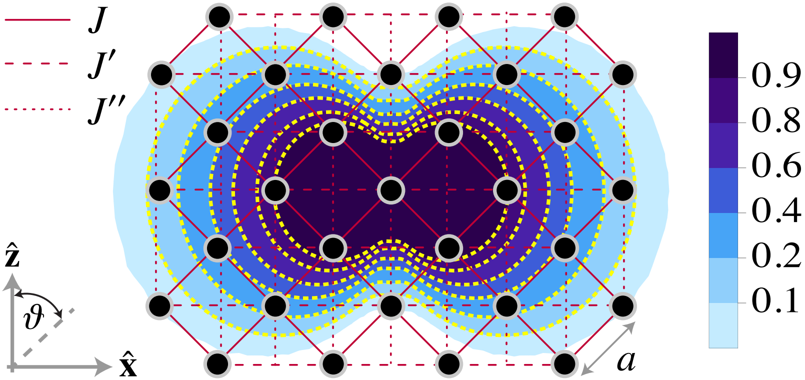
Outline

"String theory on a chip" with superconducting circuits



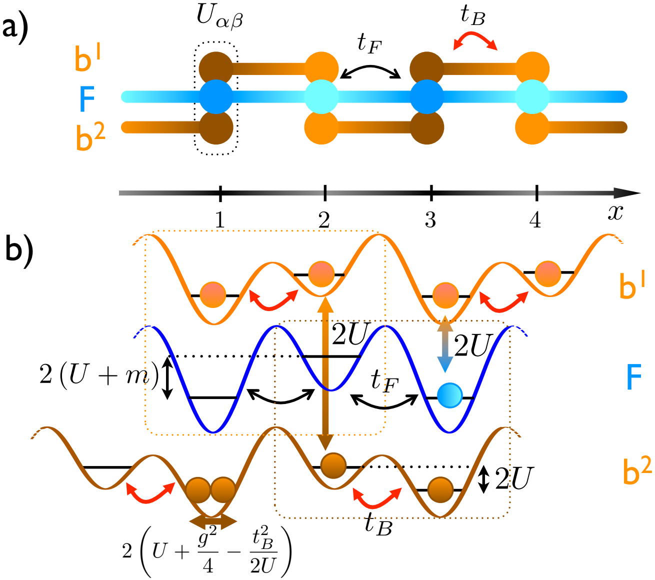
D. Marcos, P. Rabl, E. Rico, P. Zoller,
Phys. Rev. Lett. 111 (2013) 110504.
D. Marcos, P. Widmer, E. Rico, M. Hafezi, P. Rabl, UJW, P. Zoller,
Annals of Physics 351 (2014) 634.

P-state excited Rydberg atoms in an optical lattice



A. G. Glaetzle, M. Dalmonte, R. Nath, I. Rousochatzakis, R. Moessner, P. Zoller, Phys. Rev. X4 (2014) 041037.

Optical lattice with Bose-Fermi mixture of ultra-cold atoms

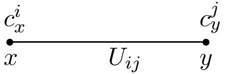


D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

Fermionic rishons at the two ends of a non-Abelian link

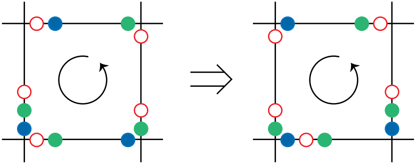
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

Rishon representation of link algebra

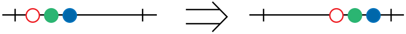


$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a “rishon abacus” implemented with ultra-cold atoms be used as a quantum simulator?

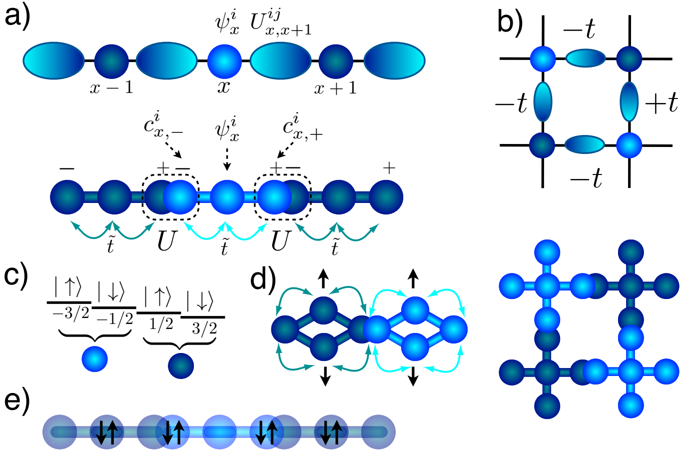


Tr Up



det $U_{x,\mu}$

Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin



D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

Outline

Extending our brain with computing machines

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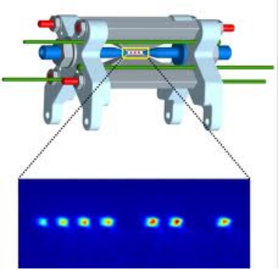
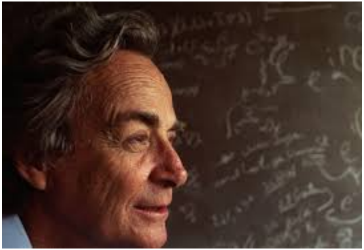


Seek inspiration from a great mind



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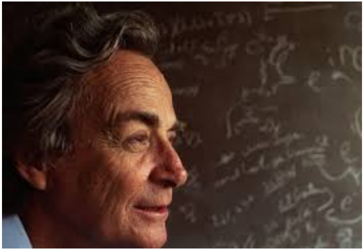
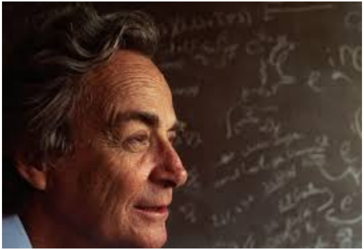


The Future of Physics (1961): “We live in a heroic, a unique and wonderful age of excitement. It’s going to be looked at with great jealousy in the ages to come. I believe that fundamental physics has a finite lifetime. It has a while to go. I take advantage of the fact that I live at the right age, but I don’t think it will go on for a thousand years.”

Seek inspiration from a great mind



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The Value of Science (1955): "We are at the very beginning of time for the human race. It is not unreasonable that we grapple with problems. But there are tens of thousands of years in the future. Our responsibility is to do what we can, learn what we can, improve the solutions, and pass them on."